**MTH 225 Winter 2025 -- Application/Analysis Exam 1**

**Version 2**

**Multiple Choice**

For each of the questions in this part, CIRCLE the ONE answer (or just the letter in front of the answer) that is most correct. No explanations are needed.

1. What is the primary purpose of the Euclidean Algorithm?
   1. To find the product of two integers
   2. To determine the least common multiple (LCM)
   3. To compute the greatest common divisor (GCD)
   4. To solve linear equations
2. In the Euclidean Algorithm, if a = 196 and b = 28, what is the first step?
   1. Compute 196 - 28
   2. Compute 196 % 28
   3. Compute 28 % 196
   4. Compute the quotient of 196 divided by 28
3. In a conditional statement "If p, then q", what is q called?
   1. Conclusion
   2. Hypothesis
   3. Consequent
   4. Proposition
4. Which logical connective is used to denote "or"?
   1. ∨
   2. ⇒
   3. ∧
   4. ¬
5. If a logical argument is valid, it means that
   1. The conclusion is never False
   2. The premises are never all False at the same time
   3. The conclusion is True in all cases where the premises are all True
   4. There is at least one condition where all the premises are True and the conclusion is True
6. The number of rows (other than the header) in a truth table for a proposition that uses five different variables is
   1. 5
   2. 10
   3. 25
   4. 32
7. What is the negation of the statement "There exists a real number x such that x > 0"?
   1. For all real numbers x, x ≤ 0.
   2. For all real numbers x, x = 0.
   3. There exists a real number x such that x ≤ 0.
   4. There exists a real number x such that x > 0.
8. Consider the predicate P(x,y): "x is a proper divisor of y". (This means that x divides y evenly, but x is not equal to y.) The domain is all positive integers. Which of the following correctly expresses "*Every number greater than 1 has at least one proper divisor*"?
   1. ∀x > 1, ∃y(P(y,x))
   2. ∀x∀y(x > 1 → P(y,x))
   3. ∃x > 1, ∀y(P(y,x))
   4. ∀x∃y(x > 1 ∧ P(x,y))
9. Consider the proposition: For all integers n ≥ 4, . The predicate involved in this proposition is
   1. For all integers n ≥ 4,
   2. None of these
10. Consider the proposition: For all integers n ≥ 4, . Suppose we wanted to prove this using mathematical induction. To prove the base case, we would need to
    1. Show by direct demonstration that 02 is less than or equal to 20
    2. Show by direct demonstration that 12 is less than or equal to 21
    3. Show by direct demonstration that 42 is less than or equal to 24
    4. Show by direct demonstration that 24 is less than or equal to 42
    5. None of these

**Problem Group 1: Computation**

Choose **ONE and ONLY ONE** of the problems below and give a complete, clear, and correct solution in the space provided. Be sure to show all work and explain your reasoning.

1. Convert the number 589673 from base 10 to base 35 using the base conversion algorithm. For symbols, use ordinary 0-9 for zero through nine and lower-case letters a through z for the numbers 10 through 35.
2. Using the “repeated squaring” algorithm from Application/Analysis Homework 2, find the last two digits of the number 397 . The number 97 in binary form is 1100001.

**Problem Group 2: Algorithms and proofs**

Choose **ONE and ONLY ONE** of the problems below and give a complete, clear, and correct solution in the space provided. Be sure to show all work and explain your reasoning.

NOTE: Extra space for this problem group is given on the next page.

1. Below is a recursive Python function called “mpower”, first seen on Application/Analysis Homework 4. As a reminder, “//” means integer division or “floor division” where we divide but only keep the quotient, for example 16 // 5 = 3. Manually work through this code to find mpower(3, 25, 100).

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1. Consider the proposition: **For all natural numbers *n*, 6n is a multiple of 5**. Below is a proposed proof for this proposition using mathematical induction. What, if anything, is wrong with the proof? If there are no major issues, say so. If there are no *major* issues but it does have one or more minor ones, then list those and explain why they are issues, and why they are minor. If there *are* major issues with being clear, correct, or complete, then list those and explain why they are issues, and why they are major.

**Proof:** We prove this with mathematical induction. The base case is when n = 0, so we will first show 60 is a multiple of 5. Well, 60 = 1 and 1 is a multiple of 5, namely .

Now assume that for some positive integer k, 6k is a multiple of 5. We want to show that 6k+1 is a multiple of 5. Since 6k is a multiple of 5 by assumption, we can write it as 6k = 5x for some integer x. Now multiply both sides of the equation by 6, and we have 6k+1 = 6(5x). Rearranging the right side (using the “commutative property” and “associative property” of multiplication) we get 5(6x) on the right, so it’s a multiple of 5. Therefore 6k+1 is a multiple of 5 as desired.

(Extra space for Problem Group 2)

**Problem Group 3: Logic**

Choose ONE and ONLY ONE of the problems below and give a complete, clear, and correct solution in the space provided. Be sure to show all work and explain your reasoning. (Note, the second problem has two subparts; if you choose that one, you must do both subparts.)

1. Use a truth table to determine whether the following argument is valid or invalid: The premises are and , and the conclusion is . Be sure to clearly state whether the argument is valid or invalid, and how you know this from the truth table. Be sure to show all intermediate columns in the truth table.
2. Use truth tables to determine whether each pair of statements is logically equivalent or not. Be sure to show all intermediate columns.
   1. and
   2. and